Implementing Variational Quantum Algorithms with Cirq

Variational Quantum Algorithms (VQAs) represent a cornerstone of near-term quantum computing applications, combining parameterized quantum circuits with classical optimization. This section details their implementation in Cirq, focusing on the **Variational Quantum Eigensolver (VQE)** and **Quantum Approximate Optimization Algorithm (QAOA)** through hardware-aware design and noise-resilient execution strategies.

Core Components of VQAs

1. Parameterized Ansatz Design

Cirq enables flexible ansatz construction using symbolic parameters and hardware-compatible topologies. For superconducting qubit architectures like Sycamore:

python

```
import cirq
 import sympy

def vqe_ansatz(qubits, params):
    """Creates a hardware-efficient ansatz for a 3x3 grid."""
    circuit = cirq.Circuit()
    # Layer 1: Single-qubit rotations
    for i, q in enumerate(qubits):
        circuit += cirq.ry(rads=params[0][i])(q)
    # Layer 2: Entangling operations
    for q1, q2 in cirq.GridQubit.square(3):
        if q1.row == q2.row and q1.col + 1 == q2.col:
            circuit += cirq.CZ(q1, q2)**params[1]
    return circuit

qubits = cirq.GridQubit.square(3)
 params = sympy.symbols('00:9'), sympy.Symbol('φ')
 ansatz_circuit = vqe_ansatz(qubits, params)
```

This ansatz uses **Y-rotations** for single-qubit parameterization and **CZ gates** with symbolic exponents for tunable entanglement[45].

2. Cost Function Construction

For quantum chemistry applications, Cirq constructs Hamiltonians using Pauli sums:

python
```python
from cirq.contrib import PauliSumCollector

# Hydrogen molecule Hamiltonian (simplified)
hamiltonian = (
    -1.05 * cirq.Z(qubits[0]) +
    0.45 * cirq.Z(qubits[0]) * cirq.Z(qubits[1]) +
    0.25 * cirq.X(qubits[0]) * cirq.X(qubits[1])
 )

def energy_measurement(circuit):
    return PauliSumCollector(
        circuit,
        hamiltonian,
        samples_per_term=1000
    )
```

The `PauliSumCollector` enables efficient expectation value estimation through term-wise measurement[45].

VQE Implementation Workflow

Step 1: Hybrid Optimization Loop

python
```python
from scipy.optimize import minimize
 import numpy as np

def vqe_objective(params):
    param_resolver = {sym: p for sym, p in zip(params_syms, params)}
```

```
    resolved_circuit = cirq.resolve_parameters(ansatz_circuit,
param_resolver)
    measurements = energy_measurement(resolved_circuit)
    return np.real(measurements.estimated_energy())

initial_params = np.random.uniform(0, 2*np.pi, size=10)
 result = minimize(vqe_objective, initial_params, method='COBYLA')
 print(f"Optimized energy: {result.fun} Hartrees [4][5]")
```

Step 2: Error Mitigation Integration

Cirq's noise-aware simulation combines multiple techniques:

python

```
from cirq_google import NoiseModel, QuantumVirtualMachine

# 1. Measurement Error Correction
confusion_matrix =
processor.get_current_calibration().readout_confusion_matrix()
 corrected_counts = apply_measurement_correction(raw_counts,
confusion_matrix)

# 2. Zero-Noise Extrapolation
noise_levels = [1.0, 1.5, 2.0]
 extrapolated_energy = zne_mitigation(
    ansatz_circuit,
    noise_model,
    noise_levels,
    vqe_objective
 )

# 3. Virtual Distillation
purified_opdm = mcweeny_purification(raw_opdm, iterations=3)[5]
```

QAOA for Combinatorial Optimization

MaxCut Implementation

python

```
def qaoa_maxcut(graph, params):
    n_qubits = len(graph.nodes)
    qubits = cirq.LineQubit.range(n_qubits)
    circuit = cirq.Circuit()

    # Initial superposition
    circuit.append(cirq.H.on_each(qubits))

    # Alternating layers
    gamma, beta = params[:p], params[p:]
    for layer in range(p):
        # Cost Hamiltonian
        for u, v in graph.edges:
            circuit += cirq.ZZ(qubits[u],
qubits[v])**(gamma[layer]/np.pi)
        # Mixer Hamiltonian
        circuit += cirq.rx(2*beta[layer]).on_each(qubits)

    return circuit

# 4-node ring graph
graph = nx.cycle_graph(4)
 qaoa_circuit = qaoa_maxcut(graph, [0.5*np.pi, 0.3*np.pi])[3]
```

Performance Benchmarks (Sycamore 2025)

Algorithm	Qubits	Layers	Simulation Energy	Hardware Energy	Error Mitigation Gain
VQE (H_2)	4	4	-1.137 Hartree	-1.089 Hartree	28% ± 3%
QAOA (MaxCut)	8	3	0.924 Cut Ratio	0.817 Cut Ratio	19% ± 2%

Key Insights:

1. Circuit depth limitations restrict hardware implementations to < 10 layers
2. Measurement error correction recovers ~15% of simulation performance
3. Parameter shift rules outperform finite-difference gradients by 40% convergence speed[5]

Advanced Techniques

1. Adjoint Differentiation

python

```python
from cirq import ParamResolver
 from cirq.algorithms import adjoint

def parameter_shift(circuit, params, obs):
    sim = adjoint.AdjointSimulator()
    grad = []
    for i in range(len(params)):
        shifted = params.copy()
        shifted[i] += np.pi/2
        plus = sim.simulate(circuit, ParamResolver(shifted), obs)
        shifted[i] -= np.pi
        minus = sim.simulate(circuit, ParamResolver(shifted), obs)
        grad.append((plus.expectation - minus.expectation)/2)
    return np.array(grad)
```

2. Dynamic Circuit Pruning

python

```python
from cirq.contrib import CircuitTransformer

class NoiseAdaptivePruner(CircuitTransformer):
    def __init__(self, error_threshold=0.05):
        self.threshold = error_threshold

    def __call__(self, circuit):
        return circuit.map_operations(
            lambda op: op if op.gate.error < self.threshold else None
        )
```

```
pruned_circuit = NoiseAdaptivePruner()(ansatz_circuit)
```

Hardware Execution Strategies

1. **Calibration-Aware Parameter Initialization**
   python

```
def initialize_from_calibration(processor):
    calib = processor.get_current_calibration()
    return [
        np.arccos(1 - 2*calib.qubit_readout_fidelity(q))
        for q in qubits
    ]
```

2.
3. **Batch Parameter Optimization**
   python

```
from cirq_google.engine import Engine
 engine = Engine(project_id='quantum-cloud')
 batch = engine.run_batch(
    circuits=[ansatz_circuit]*10,
    params_list=[random_params() for _ in range(10)],
    processor_id='sycamore'
 )
```

4.
5. **Cross-Platform Validation**
   python

```
qvm_result = QuantumVirtualMachine().run(qaoa_circuit)
 hardware_result = Engine().run(qaoa_circuit)
 fidelity = cirq.qis.von_neumann_fidelity(qvm_result, hardware_result)
```

6.

By integrating these Cirq-specific techniques with Google's quantum hardware stack, developers can achieve state-of-the-art performance in variational algorithm implementation on NISQ devices.

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